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Sonic Boom of Hypersonic Vehicles

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Introduction

THE phenomenon of sonic boom has received considerable attention in the past decade. Generally, the sonic boom of a supersonic aircraft can be predicted accurately, at least in the far-field, by the current sonic boom theory which is based on the well-known supersonic area rule and Whitham's supersonic projectile theory. The supersonic area rule approximates three-dimensional configurations to equivalent bodies of revolution, whereas Whitham's theory describes the far-field behavior of the flow over these bodies of revolution. The sonic boom of a hypersonic vehicle, however, cannot be predicted by the current sonic boom theory. A general "hypersonic area rule" has not yet been developed to simplify vehicle configurations, and moreover, Whitham's theory which is equivalent to a geometrical acoustic approach is not generally applicable in the hypersonic flowfield in which nonlinear effects are dominant. Indeed recent experiments have shown deficiencies of the current sonic boom theory at high Mach numbers.[1]

In this Note, we shall be concerned with the far-field flow behavior of a vehicle at hypersonic speeds. Particularly an approximate method for determining the sonic boom strength, position and the positive phase duration is presented.

Equivalence Principle

It is physically known that, at hypersonic speeds, a body leaves a long wake behind it and, at a distance sufficiently far from this body, the flowfield becomes axisymmetric. Although no studies have yet shown how a nonaxisymmetric disturbance would approach to an axisymmetric one in hypersonic flows, the flow disturbances in the far-field, to a first approximation, may be considered as those produced by an equivalent body of revolution having experienced the same total drag as the actual body. Thus we may consider a steady inviscid hypersonic flow about blunt-nosed axisymmetric slender bodies, in particular those configurations having long cylindrical after-bodies representative of the viscous wake.

It is well known in the hypersonic flow theory[2] that the steady, inviscid, hypersonic flow over slender bodies can be treated by the hypersonic-small-disturbance theory. If the streamwise coordinate x is considered as the time t, the equations in the hypersonic-small-disturbance theory are identical with the full (exact) equations for a corresponding unsteady flow in one less space variable. Consequently, the hypersonic flow over a blunted-nosed axisymmetric slender body having a long cylindrical after-body may be considered as the steady-state analogy to the constant-energy cylindrical blast wave problem.[3] The nose drag in the steady problem is equivalent to the finite energy which is instantaneously released in the unsteady problem.

In the far-field of a slender body, the flow is only slightly disturbed and shock waves are already weak even in the hypersonic flow. The governing equations for the steady hypersonic flow in the far-field can be linearized. The linearized equations for the steady hypersonic flow with the streamwise coordinate x replaced by $Ut(1-M^{-2}) \approx Ut$ are exactly analogous to the unsteady two-dimensional acoustic equations.[4] (U and M are the freestream velocity and Mach number, respectively.) Therefore, the equivalence of the steady three-dimensional flow with the unsteady two-dimensional flow exists in the entire hypersonic flowfield over slender bodies.

Unsteady Cylindrical Wave

Solutions to the unsteady two-dimensional problem with cylindrical symmetry have been found only in the limiting cases of very strong[3] and very weak shock wave.[5] Attempts to extend these solutions to the intermediate strength of shock waves by using series expansion have not been very successful. Recently, Plooster[6] has obtained several numerical solutions for cylindrical shock waves from line sources. His computations extend blast wave solutions well into the weak shock region. For the purpose of determining sonic booms, Plooster's lowest data of shock strength 0.1 are still too high to be directly applicable. (For example, the sonic booms for SST are about the order of 10^{-3}.) Hence, we shall extrapolate Plooster's numerical data for the case of line source idea gas to the weak shock region.

The propagation of weak shock waves can be described by the weak shock theory based on the geometrical acoustics.[5] By calculating the rays and ray-tube areas, the pressure disturbance at any point can be determined from a given initial disturbance. Now we assume that the weak shock theory may be matching directly with Plooster's data for the lowest shock overpressure $[(\Delta p/p_o)_s \simeq 0.1]$ signature. This assumption can be justified quantitatively and qualitatively.[4] Following the procedure described above or used in the current sonic boom calculation, complete pressure signatures can be obtained at any distance farther afield.

Since the positive phase overpressure signature of Plooster $[(\Delta p/p_o)_s \simeq 0.1]$ is linearly varied with the space coordinate, this linear variation will remain in the farther afield. The positive overpressure signature, which is important in the calculation of the sonic boom impact, can be specified by the shock overpressure $(\Delta p/p_o)_s$, shock location t_s vs r_s and the duration of the positive phase L. By matching Plooster's data, $[(\Delta p/p_o)_s = 0.1]$, with the weak shock theory

$$(\Delta p/p_o)_s = 0.343/\lambda_s^{1/2}(\lambda_s^{1/2}-0.908)^{1/2} \tag{1}$$

$$\tau_s = \lambda_s - 0.160 - 0.588\,(\lambda_s^{1/2}-0.908)^{1/2} \tag{2}$$

$$l = 0.588\,(\lambda_s^{1/2}-0.908)^{1/2} \tag{3}$$

Here $\tau = C_o t/R_o$, $\lambda = r/R_o$ and $l = L/R_o$, with C_o being the sound speed of the undisturbed flow, R_o a characteristic radius $(E/b\gamma p_o)^{1/2}$, E the energy released per unit length of the line source and $b = 0.985$ for air with $\gamma = 1.4$[7].

The accuracy of the above extrapolation formula, Eqs. (1–3) can be examined. By matching the weak shock theory with Plooster's signature of shock overpressure 0.19, which is 90% higher than that for Eqs. (1–3), we found that the new results differ from Eqs. (1–3) by less than one percent. This justifies our assumption that the weak shock theory can be directly matched with Plooster's lowest shock overpressure signature $[(\Delta p/p_o)_s = 0.1]$.

Sonic Boom Signatures

After having obtained a solution for the unsteady cylindrical wave, we may find the flow pattern of a hypersonic blunt-nosed axisymmetric slender body by proper changes of the variables. By the use of the blast wave analogy, the energy released per unit length of line source is replaced by the total drag of the body and, in turn, can be related to the drag coefficient C_D, dimension d and freestream conditions ρ_o, U of the body,

$$E = D = (\pi/8)(C_D d^2 \rho_o U^2) \tag{4}$$

By using the equivalence principle, the nondimensional variables τ and λ may be expressed in the space variables x and r, respectively, with the parameters M, C_D and d,

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$$x/d/(M^2C_D{}^{1/2}) = (\pi/8b)^{1/2}\tau, \quad r/d/(MC_D{}^{1/2}) = (\pi/8b)^{1/2}\lambda \tag{5}$$

Thus, the complete steady flow pattern can be obtained from the unsteady cylindrical wave pattern.

In the far-field (weak shock region), since the overpressure of the positive phase varies linearly with the space variables, the positive phase overpressure, which has practical importance, can be described by the shock overpressure $(\Delta p/p_o)_s$, the shock position x_s vs r_s, and the duration L_r (in the radial direction) or L_x (in the axial direction) of the positive phase signature. The explicit and useful relations can be readily obtained from Eqs. (1–3): the shock position,

$$\frac{x_s/d}{M^2C_D{}^{1/2}} = \frac{r_s/d}{MC_D{}^{1/2}} - 0.101 - 0.416\left[\frac{(r_s/d)^{1/2}}{M^{1/2}C_D{}^{1/4}} - 0.721\right]^{1/2} \tag{6}$$

the shock overpressure

$$\left(\frac{\Delta p}{p_o}\right)_s = 0.306\,\frac{MC_D{}^{1/2}}{r_s/d}\left[\frac{(r_s/d)^{1/2}}{M^{1/2}C_D{}^{1/4}} - 0.721\right]^{-1/2} \tag{7}$$

and the durations of the positive phase

$$\frac{L_x/d}{M^2C_D{}^{1/2}} = \frac{L_r/d}{MC_D{}^{1/2}} = 0.416\left[\frac{(r_s/d)^{1/2}}{M^{1/2}C_D{}^{1/4}} - 0.721\right]^{1/2} \tag{8}$$

The extrapolation expressions, Eqs. (6–8), and the original Plooster's numerical data valid in the near field of the hypersonic body are plotted in Fig. 1.

Conclusions

An approximate method is developed for determining the far-field flow pattern of hypersonic bodies. For our special interests on sonic booms, the sonic boom shock overpressure, locations and the positive phase duration are expressed explicitly in terms of the Mach number, drag coefficient, and dimension of hypersonic vehicles.

Numerical examples have been carried out and reported in Ref. 4 for several steady level flights and for a typical glide trajectory of space shuttles. Figure 2 shows the sonic boom positive phase signatures of a hypersonic vehicle during its steady level flights. $C_D = 0.914$ at several altitudes and Mach numbers are assumed. The general behavior of the sonic boom during steady level flights is consistent with that of the sonic boom of supersonic vehicles. That is the larger the Mach number, the larger the drag coefficient, or the lower the altitude, the larger are the sonic boom overpressure, positive duration, and sonic boom impact. For a typical glide trajector of space shuttles, approximate results indicate that the sonic boom may be felt along the ground trace of space shuttles.

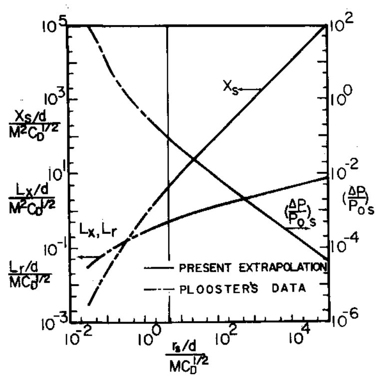

Fig. 1 Sonic boom overpressure $(\Delta p/p_o)_s$, position x_s, and positive phase duration L_r or L_x vs shock radius r_s.

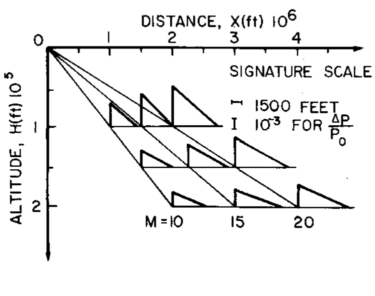

Fig. 2 Sonic boom positive phase signatures ($C_D = 0.914$).

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Structural Optimization in Random Vibration Environment

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Introduction

ALL flight vehicles operate in random vibration environment. A realistic formulation of the optimum design problem for flight vehicle structures must, therefore, contain probabilitsic constraints on the dynamic response of the system. In this note the optimization problem in random vibration environment is stated and transformed to a standard nonlinear programming problem. Two simple examples with weight and expected rate of fatigue damage as objective functions are presented to illustrate the application of the proposed formulation.

Optimization Problem

A typical optimization problem in random vibration environment can be stated as: Minimize

$$W(\bar{D}) \tag{1}$$

subject to

$$P\left[\bigcup_{\substack{i=1\\ a\le t\le b}}^{k}\left\{s_i(\bar{x}(\bar{D},t)) \ge r_i\right\}\right]_j \le [p_f]_j \qquad j = 1,2,\ldots,m \tag{2}$$

$$s_j(\bar{D}) \le r_j \qquad j = m+1,\ldots,n \tag{3}$$

and

$$f_i^{(l)} \le f_i \le f_i^{(u)} \qquad i = 1,2,\ldots,p \tag{4}$$

where $W(\bar{D})$ is the objective function, $\bar{D}$ is the vector of the design variables, $\bar{x}(\bar{D},t)$ represents the dynamic response of the system, $s_i[\bar{x}(\bar{D},t)]$ is a response function, r_i is a deterministic or random constraint on s_i, $[p_f]_j$ denotes the specified upper bound on the probability of failure in mode j, f_i is the ith natural frequency of the system and $f_i^{(l)}$ and $f_i^{(u)}$ denote the lower and upper bounds on the frequency.

The dynamic response of a large class of problems can be treated as a stationary random process. In such cases the mean

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